Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Spatial Decay in the Response of Damped Periodic Beam

R. VAICAITIS*
Columbia University, N. Y.
K. DOI†
Mitsubishi Heavy Industries, Ltd., Tokyo, Japan
Y. K. LIN‡
University of Illinois, Urbana, Ill.

BY a periodic structure we refer to the type of construction frequently used in aerospace and marine engineering where the interior of a structure forms an array of connected identical units. The fuselage of a flight vehicle and the hull of a submarine are typical examples. Such structures are often large, and it may not be economically feasible to conduct experimental tests on the prototypes. Thus, earlier experiments on aircraft panel response to jet or boundary-layer noise, for example, were performed on single isolated panels.

It has been pointed out by Lin[1,2] that single panels are not representative of a group of interconnected panels. In particular, the response spectral density of an isolated panel to noise excitation is characterized by well-separated sharp peaks whereas the response spectrum of a multipanel system under the same excitation shows groups of peaks closely clustered within separate frequency bands. This peak grouping is not unique for periodic structures. The same phenomenon is observed when the panel size varies from one span to another.

Although the need for a multipanel specimen for laboratory testing is now generally recognized, the choice of specimen size remains an arbitrary one. The fundamental question, "How many panels must be included in a specimen in order that the specimen is dynamically representative of an actual structure?", has not been answered. The present study was motivated by the need for a rational answer to this question.

It is well known that response of a structure to an impulsive excitation decays with time since damping is always present in an actual structure. However, damping gives rise not only to temporal decay but also to spatial decay in the response. Thus, if excitation is applied within a certain panel on an aircraft fuselage, the effect of excitation will be felt strongly by the immediate adjacent panels, and this effect will be felt less and less for more distant panels. If the effect of an excitation is negligible at N panels away in one direction and M panels away in another direction, then in view of the reciprocal theorem of a linear system, it will be adequate to use a specimen consisting of $2N \times 2M$ panels for the study of dynamic behavior of a panel near the center of the structure.

As an initial step we choose an infinitely long Euler-Bernoulli beam periodically supported by elastic springs for the present study. The objective is to determine the distance, in terms of number of periodic units, required for a steady-state sinusoidal response to decay to a negligible amplitude. The results should be a valuable guide for the future similar study of two-dimensional panel arrays.

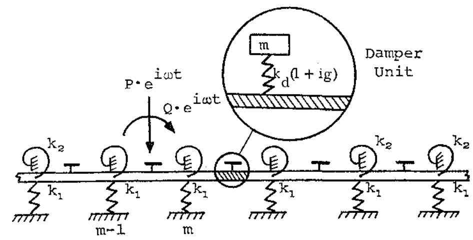

Fig. 1 Periodic beam on elastic supports.

As shown in Fig. 1 the beam is prismatic, made of homogeneous and isotropic material, and supported at uniform intervals by identical elastic springs. A tuned damper of the type investigated by Jones, Henderson and Bruns[3] is attached to each span and is identically positioned relative to the supports in each span. Let sinusoidal excitations such as $Pe^{i\omega t}$ and $Qe^{i\omega t}$ be located between supports $m-1$ and m. The steady-state response at different locations on the structure can be simply related by use of transfer matrices[4–6]. We shall be concerned only with response at the points of demarcation for neighboring periodic units (to be called periodic stations). Specifically, if excitations are not present between two periodic stations n and $n+1$, then the response state vectors at these two stations are related by [5,6]

$$\mathbf{Z}_{n+1} = T\mathbf{Z}_n \tag{1}$$

where, in the present case, $\mathbf{Z}$ is a four-dimensional state vector whose components are the complex amplitudes of deflection, slope, moment and shear, denoted by δ, ϕ, M, V, and where T is the transfer matrix associated with the structural element, connecting stations n and $n+1$. For the construction of this matrix the reader is referred to Ref. 6.

We note some interesting properties of a transfer matrix[6]: 1) the determinant of a transfer matrix is unity, 2) inversion can be accomplished by rearranging the elements and changing some of their signs, 3) the eigenvalues are in reciprocal pairs, 4) if the structural element represented by a transfer matrix is symmetrical, then by a suitable choice of the order and sign convention for the components of state vector $\mathbf{Z}$, the transfer matrix can be made symmetrical about its cross diagonal.

Since the beam is infinitely long the relation between response state vectors can also be expressed as follows:

$$\mathbf{Z}_{n+1} = e^{i\theta}\mathbf{Z}_n \tag{2§}$$

When θ is purely real Eq. (2) simply states that the state vector at $n+1$ lags (or leads) the state vector at n by a phase angle θ. However, purely real θ values are possible only in the case of undamped free wave motion and at a frequency falling within one of the natural frequency bands[1,9].

Comparing Eqs. (1) and (2) it is clear that $e^{i\theta}$ is an eigenvalue of T. This suggests a simple way to calculate θ and to study its properties. Recall that eigenvalues of T are

Submitted April 7, 1971; revision received August 19, 1971. The work was supported by U. S. Air Force Materials Laboratory under Contract F33615-67-C-1190.

* Assistant Professor of Civil Engineering and Engineering Mechanics. Associate Member AIAA.
† Engineer.
‡ Professor, Aeronautical and Astronautical Engineering. Member AIAA.

§ Mead[7,8] has used a propagation constant μ instead of $i\theta$ in his work.

reciprocal pairs. Thus, denoting these eigenvalues by exp $(\pm i\theta_1)$ and $\exp(\pm i\theta_2)$, we can show that[6]

$$\begin{matrix}\cos\theta_1 \\ \cos\theta_2\end{matrix} = \frac{K_1}{2} \pm \left(\frac{K_1^2}{4} - \frac{K_2 - 1}{2}\right)^{\frac{1}{2}} \tag{3}$$

where K_1 and K_2 are related to elements t_{ij} of the transfer matrix T as

$$K_1 = \tfrac{1}{2}\sum_{i=1}^{4} t_{ii}$$

$$K_2 = \tfrac{1}{4}\sum_{i=1}^{4}\sum_{\substack{j=1\\ i\neq j}}^{4} (t_{ii}t_{jj} - t_{ij}t_{ji}) \tag{4}$$

Since we are only interested in damped structures, neither θ_1 nor θ_2 can be purely real at any frequency. Without loss of generality let the imaginary parts of θ_1 and θ_2 be both positive. Then each reciprocal pair, θ_j and $-\theta_j$, characterize the same type of wave motion: θ_j associated with a right-going wave and $-\theta_j$ associated with a left-going wave. Under an excitation between supports $m-1$ and m, as illustrated in Fig. 1, response to the right of m is superposition of two right-going waves, and response to the left of $m-1$ is that of two left-going waves, and attenuation of response can be determined from the imaginary parts of θ_1 and θ_2.

Let the excitations be located left of periodic station n and let the state vector at station n be separated into

$$\mathbf{Z}_n = \mathbf{Z}_n^{(1)} + \mathbf{Z}_n^{(2)} \tag{5}$$

where the two component vectors are associated with θ_1 and θ_2, respectively. Thus, considering both types of wave, we have, from Eq. (2)

$$\mathbf{Z}_{n+M} = e^{iM\theta_1}\mathbf{Z}_n^{(1)} + e^{iM\theta_2}\mathbf{Z}_n^{(2)} \tag{6}$$

We define a spatial decay ratio as follows:

$$r(M) = \frac{\|\mathbf{Z}_{n+M}^{(1)}\| + \|\mathbf{Z}_{n+M}^{(2)}\|}{\|\mathbf{Z}_n^{(1)}\| + \|\mathbf{Z}_n^{(2)}\|} \tag{7}$$

where $\|\ \|$ denotes a simple norm of a vector; that is, the sum of the absolute values of its components. In the present case $\|Z\| = |\delta| + |\phi| + |M| + |V|$. The exact value of $r(M)$ as given in Eq. (7) is difficult to evaluate. It depends on the relative contribution of the two types of waves which, in turn, depends on the exact location of an excitation. However, it is simple to find an upper bound to Eq. (7). Write

$$r(M) = \exp(-\psi_s M)\,\frac{\|\mathbf{Z}_n^{(s)}\| + \exp[(-\psi_\ell + \psi_s)M]\|Z_n^{(\ell)}\|}{\|Z_n^{(s)}\| + \|Z_n^{(\ell)}\|} \tag{8}$$

where $\psi_s = \min(\psi_1, \psi_2)$, $\psi_l = \max(\psi_1, \psi_2)$ and the superscripts (s) and (ℓ) denote the vectors associated with ψ_s and ψ_ℓ, respectively. An upper bound of $r(M)$ may then be obtained from

$$r(M) \leq \exp(-\psi_s M) \tag{9}$$

The equality sign corresponds to the case $\psi_1 = \psi_2$.

Numerical Examples and Discussion

Numerical examples were worked out using a criterion for negligible response $r(M) \leq 0.01$. The beam chosen for the calculation is made of aluminum (Young's modulus $= 10.5 \times 10^6$ psi, weight 0.101 lb/in.^{3}, and is 0.04 in. thick and 1 in. wide. The elastic supports are 8.2 in. apart; each is characterized by a translational spring constant $k_1 = 1100$ lb/in. and a rotational spring constant $k_2 = 180$ in.-lb/rad. Structural damping of the beam material and the elastic supports is accounted for by multiplying complex factors $(1 + ig_{sd})$, $(1 + ig_{des})$ and $(1 + ig_{res})$ to E, k_1 and k_2, respectively. The damper unit attached at every mid-span is tuned at frequency ω_n and each has a loss factor $g = 0.2$.

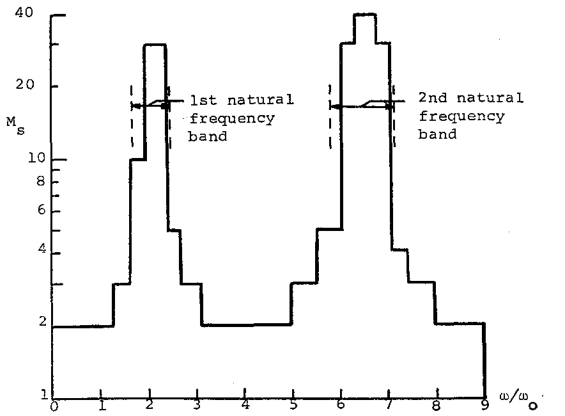

Fig. 2 Number of spans from excitation for negligible response $g = 0.2$, $g_{sd} = 0.02$, $g_{res} = 0.05$, $g_{des} = 0.05$, $\omega_n/\omega_o = 2.94$.

Typical results are shown in Figs. 2 and 3 where the abscissa is the excitation frequency normalized to the fundamental natural frequency ω_o(340 rad/sec) of a single span beam simply supported at both ends and the ordinate M_s is the smallest integer M satisfying the above criterion for negligible response. Bands of natural frequencies at which free wave propagation in the case of undamped systems may exist are also marked in the figures. It is seen that the M_s value is highly dependent on the excitation frequency and that the peak regions for M_s coincide with the natural frequency bands. One important effect of the tune dampers is the suppression of the M_s value (increasing the spatial decay rate of the response) near the tuning frequency. As can be seen by a comparison of Figs. 2 and 3, the first peak region is greatly reduced when dampers are tuned at a frequency near the middle of that region.

Although the calculations reported above pertain to a simple Euler-Bernoulli beam, some conclusions drawn from the present study are believed to apply to all periodic structures, including two-dimensional array of skin-stiffener panels. Firstly, since the rate of spatial decay in the response depends on the excitation frequency the size of a smaller specimen for laboratory testing is also determined by this frequency, or, in the case of noise loading, by the shapes of spectral density and cross-spectral density of the forcing field. When the number of spans ($2M_s$) required of a specimen is near or beyond that of the prototype, the need for using the entire structure in the experimental study is indicated. Secondly,

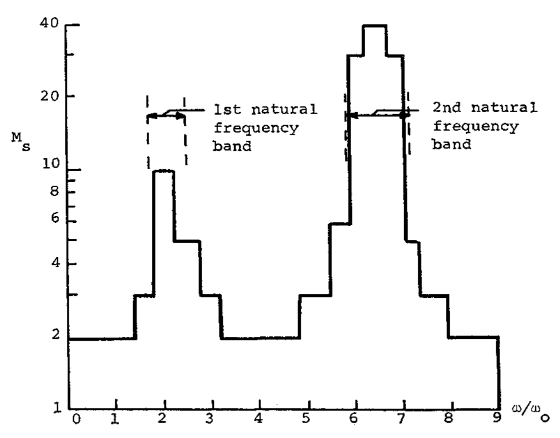

Fig. 3 Number of spans from excitation for negligible response $g_{sd} = 0.02$, $g_{res} = 0.05$, $g_{des} = 0.05$, $\omega_n/\omega_o = 2.06$.

we have demonstrated the usefulness of the type of dampers investigated in Ref. 3 for vibration control of periodic structures. Such dampers may be tuned to suit the spectral shape of a noise environment.

References

[1] Lin, Y. K., "Free Vibration of Continuous Skin-Stringer Panels," *Journal of Applied Mechanics*, Vol. 27, No. 4, 1960, pp. 669–676.
[2] Lin, Y. K., "Stresses in Continuous Skin-Stinger Panels under Random Loading," *Journal of the Aerospace Sciences*, Vol. 29, No. 1, 1962, pp. 67–76.
[3] Jones, D. I. G., Henderson, J. P., and Bruns, G. H., "Use of Tuned Viscoelastic Dampers for Reduction of Vibrations in Aerospace Structures," presented at the 13th Annual Air Force Science and Engineering Symposium, Arnold Air Force Station, Tenn., Sept. 27–27, 1966.
[4] Pestel, E. C. and Leckie, F. A., *Matrix Methods in Elastomechanics*, McGraw-Hill, New York, N.Y., 1963, Chap. 5.
[5] Lin, Y. K. and Donaldson, B. K., "A Brief Survey of Transfer Matrix Techniques with Special Reference to the Analysis of Aircraft Panels," *Journal of Sound and Vibration*, Vol. 10, No. 1, 1969, pp. 103–143.
[6] Lin, Y. K. and McDaniel, T. J., "Dynamics of Beam Type Periodic Structures," *Journal of Engineering for Industry, Transactions of the ASME*, Vol. 91, No. 13-4, 1969, pp. 1133–1141.
[7] Mead, D. J., "Free Wave Propagation in Periodically Supported Infinite Beams," *Journal of Sound and Vibration*, Vol. 11, No. 2, 1970, pp. 181–197.
[8] Mead, D. J., "Vibration Response and Wave Propagation in Periodic Structures," Paper 70-WA/DE-3 1970, ASME.
[9] Miles, J. W., "Vibrations of Beams on Many Supports," *Journal of Engineering Mechanics Division, Proceedings of the ASCE*, Vol. 82, pp. 1–9, EM 1, 1956,
[10] Sen Gupta, G., "On the Relation between the Propagation Constant and the Transfer Matrix Used in the Analysis of Periodically Stiffened Structures," *Journal of Sound and Vibration*, Vol. 11, No. 4, 1970, pp. 483–484.
[11] Lin, Y. K., "Free Vibrations of a Continuous Beam on Elastic Supports," *International Journal of Mechanical Sciences*, Vol. 4, 1962, pp. 409–423.

A Simplified Model for Aircraft Steering Dynamics

CARL GRUBIN*
Hughes Aircraft Company, Culver City, Calif.

THIS Note describes a simplified dynamical model of an aircraft that is useful for combat simulations involving aircraft steering. Both translational and rotational equations are developed. The former are obtained by expanding the equations of motion along aircraft flight path axes. The latter, however, are obtained only from geometrical considerations and transfer functions for the lift magnitude and lift bank-angle. In this sense, the equations are simplified.

Received March 12, 1971; revision received September 10, 1971.
Index categories: Aircraft Flight Operations, Aircraft Handling, Stability and Control.
* Senior Staff Engineer, Systems Division. Associate Fellow AIAA.

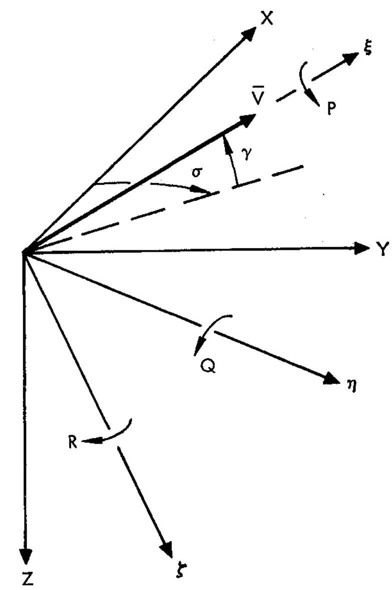

Fig. 1 Aircraft velocity vector.

Analysis

Flat Earth assumptions are made so that a local vertical, Earth-fixed reference frame is considered an inertial frame. The frame axes are X, Y, Z, where XY are in the local horizontal plane and Z is down along local vertical. Aircraft velocity vector **V** is expressed in terms of its magnitude V, and azimuth and elevation flight-path angles σ, γ respectively (Fig. 1). Angle σ is measured from the X axis to the projection of **V** in the XY plane, γ is the flight-path angle of **V** above the XY plane. Introduce axes ξ, η, ζ where ξ lies along **V**, η is perpendicular to ξ and lies in the XY plane, and ζ completes a right-handed system. (If **V** lies along $\pm Z$, then η, ζ are ill-defined. This special case is examined in the Appendix).

The forces which act on the aircraft are lift L, drag D, thrust T and gravity W (Fig. 2). L is always perpendicular to **V** and so lies in the $\eta\zeta$ plane at some angle ρ to the $-\zeta$ axis. Drag

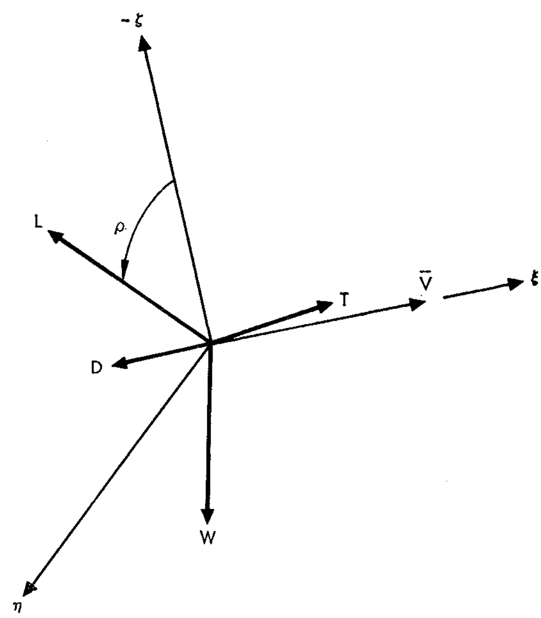

Fig. 2 Forces on aircraft.